

L^∞ TO L^p CONSTANTS FOR RIESZ PROJECTIONS

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ABSTRACT. The norm of the Riesz projection from $L^\infty(\mathbb{T}^n)$ to $L^p(\mathbb{T}^n)$ is considered. It is shown that for $n = 1$, the norm equals 1 if and only if $p \leq 4$ and that the norm behaves asymptotically as $p/(\pi e)$ when $p \rightarrow \infty$. The critical exponent p_n is the supremum of those p for which the norm equals 1. It is proved that $2 + 2/(2^n - 1) \leq p_n < 4$ for $n > 1$; it is unknown whether the critical exponent for $n = \infty$ exceeds 2.

1. INTRODUCTION

This work originated in an attempt to answer the following question: Do there exist pairs of exponents q and p with $q > p > 2$ for which the Riesz projection on the infinite torus is bounded from L^q to L^p ? This question remains open, as far as we know. The present note presents a few results of some intrinsic interest in the finite dimensional setting, giving relevant background for the original problem for the infinite torus.

Using standard multi-index notation, we write the Fourier series of a function f in $L^2(\mathbb{T}^n)$ on the n -torus \mathbb{T}^n as

$$f(\zeta) = \sum_{\alpha \in \mathbb{Z}^n} \hat{f}(\alpha) \zeta^\alpha.$$

The operator

$$P_n^+ f(\zeta) = \sum_{\alpha \in \mathbb{Z}_+^n} \hat{f}(\alpha) \zeta^\alpha$$

is the Riesz projection on \mathbb{T}^n , and, as an operator on $L^2(\mathbb{T}^n)$, it has norm 1. If we instead view P_n^+ as an operator on $L^p(\mathbb{T}^n)$ for $1 < p < \infty$, then a theorem of B. Hollenbeck and I. Verbitsky [4] says that its norm is $(\sin \frac{\pi}{p})^{-n}$.

We compute the norm $\|f\|_p$ of a function f in $L^p(\mathbb{T}^n)$ with respect to Lebesgue measure σ_n on \mathbb{T}^n , normalized such that $\sigma_n(\mathbb{T}^n) = 1$. Using this normalization, we let $\|P_n^+\|_{q,p}$ denote the norm of the operator $P_n^+ : L^q(\mathbb{T}^n) \rightarrow L^p(\mathbb{T}^n)$ for $q \geq p \geq 2$. We will restrict ourselves to computations and estimates of the norms $\|P_n^+\|_{\infty,p}$. By Hölder's inequality, $p \mapsto \|P_n^+\|_{\infty,p}$ is a continuous and nondecreasing function, and, by the theorem of Hollenbeck and Verbitsky, we have $\|P_n^+\|_{\infty,p} \leq (\sin \frac{\pi}{p})^{-n}$. Of particular interest is the number

$$p_n = \sup \{ p \geq 2 : \|P_n^+\|_{\infty,p} = 1 \},$$

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called the critical exponent of P_n^+ according to the terminology of [2]. The critical exponent p_n is well-defined since clearly $\|P_n^+\|_{\infty,2} = 1$. By continuity, we have $\|P_n^+\|_{\infty,p_n} = 1$.

We will present three theorems. The first says that the critical exponent of P_1^+ equals 4. In view of this result, one is led to ask if the precise value of $\|P_1^+\|_{\infty,p}$ can be computed also when $p > 4$ and whether we can compute or estimate the critical exponent p_n for $n > 1$. These problems are only given partial solutions: Our second theorem gives the right asymptotics for $\|P_1^+\|_{\infty,p}$ when $p \rightarrow \infty$, and our third theorem says that $2 + 2/(2^n - 1) \leq p_n < 4$ when $n > 1$.

The next three sections present these results. Section 5 contains a brief discussion of the problem for the infinite torus, while the final section discusses extensions to the setting of compact abelian groups.

2. THE CRITICAL EXPONENT OF P_1^+

Theorem 1. *The critical exponent of P_1^+ is 4.*

Proof. We write $P_1^- = I - P_1^+$ and note that $(P_1^+ f)^2 \perp (P_1^- f)^2$ whenever f is a bounded function on \mathbb{T} . Thus

$$\|P_1^+ f\|_4^4 = \|(P_1^+ f)^2\|_2^2 \leq \|(P_1^+ f)^2 - (P_1^- f)^2\|_2^2 = \|f(P_1^+ f - P_1^- f)\|_2^2 \leq \|f\|_\infty^2 \|f\|_2^2.$$

This estimate implies that $p_1 \geq 4$. To see that we also have $p_1 \leq 4$, we consider the function $f(\zeta) = (1 - \epsilon\zeta)^2 / |1 - \epsilon\zeta|^2$. We assume that $0 < \epsilon < 1/2$ and find that $P_1^+ f(\zeta) = 1 - \epsilon^2 - \epsilon\zeta$. We estimate the L^p norm of $P_1^+ f$ from the power series expansion of $(1 - \epsilon\zeta/(1 - \epsilon^2))^{p/2}$. This leads to the estimate

$$\|P_1^+ f\|_p^p = 1 + \left(\frac{p^2}{4} - p\right) \epsilon^2 + O(\epsilon^4)$$

when $\epsilon \rightarrow 0$. It follows that we may achieve $\|P_1^+ f\|_p > 1$ for every $p > 4$ by choosing ϵ sufficiently small. \square

We note that in general we have $(P_n^+ f)^2 \perp ((I - P_n^+)f)^2$ only when $n = 1$, so that the preceding proof does not work when $n > 1$.

3. ASYMPTOTIC BEHAVIOR OF $\|P_1^+\|_{\infty,p}$ WHEN $p \rightarrow \infty$

Theorem 2. *We have $\lim_{p \rightarrow \infty} p^{-1} \|P_1^+\|_{\infty,p} = (\pi e)^{-1}$.*

This theorem is a corollary of a corresponding result for the Hilbert transform (the conjugation operator), which we define as

$$Hf(\zeta) = \tilde{f}(\zeta) = -i \sum_{k \in \mathbb{Z}} \text{sign}(k) \hat{f}(k) \zeta^k.$$

By a well-known theorem of Pichorides [5], we have

$$\|H\|_{p,p} = \max \left\{ \tan \frac{\pi}{2p}, \cot \frac{\pi}{2p} \right\}.$$

The Hilbert transform maps real functions to real functions, and we write $H_{\mathbb{R}}$ when the domain is a real L^p space.

Theorem 2'. *We have $\lim_{p \rightarrow \infty} p^{-1} \|H\|_{\infty,p} = \lim_{p \rightarrow \infty} p^{-1} \|H_{\mathbb{R}}\|_{\infty,p} = 2(\pi e)^{-1}$.*

The following result of Zygmund [6, Theorem 2.11, chap. VII, vol. 1] will give an upper bound for $\|H_{\mathbb{R}}\|_{\infty, p}$.

Lemma 3. (Zygmund) *For real valued f such that $|f| \leq 1$ and $0 \leq \alpha < \pi/2$, we have*

$$\frac{1}{2\pi} \int_0^{2\pi} e^{\alpha|\tilde{f}(e^{i\theta})|} d\theta \leq \frac{2}{\cos \alpha}.$$

Proof of Theorem 2'. From Zygmund's theorem and Chebychev's inequality, we get that for real valued f with $|f| \leq 1$ we have

$$\sigma_1 \left(\{ \zeta : |\tilde{f}(\zeta)| > \lambda \} \right) \leq \frac{2}{\cos \alpha} e^{-\alpha\lambda},$$

and thus

$$\|\tilde{f}\|_p^p = p \int_0^\infty \lambda^{p-1} \sigma_1 \left(\{ \zeta : |\tilde{f}(\zeta)| > \lambda \} \right) d\lambda \leq \frac{2p}{\alpha^p \cos \alpha} \Gamma(p+1).$$

Now Stirling's formula implies that

$$\lim_{p \rightarrow \infty} \frac{1}{p} \|H_{\mathbb{R}}\|_{\infty, p} \leq \frac{2}{\pi e}.$$

To prove the reverse inequality, we consider the function

$$f(e^{i\theta}) = \arg(1 - e^{i\theta}) = \begin{cases} \theta/2 - \pi/2, & 0 \leq \theta \leq \pi \\ \theta/2 + \pi/2, & -\pi < \theta < 0. \end{cases}$$

We have $H(\arg(1 - e^{i\theta})) = -\log|1 - e^{i\theta}|$ and

$$\|Hf\|_p \leq \|H_{\mathbb{R}}\|_{\infty, p} \|\arg(1 - e^{i\theta})\|_\infty = \|H_{\mathbb{R}}\|_{\infty, p} \frac{\pi}{2}.$$

Since

$$\|\log|1 - e^{i\theta}|\|_p^p = \frac{1}{\pi} \int_0^\pi \left| \log(2 \sin \frac{\theta}{2}) \right|^p d\theta \geq \frac{1}{\pi} \int_0^1 |\log \theta|^p d\theta = \frac{1}{\pi} \Gamma(p+1),$$

it follows that

$$\frac{2}{\pi e} = \lim_{p \rightarrow \infty} \frac{2}{\pi p} \|Hf\|_p \leq \lim_{p \rightarrow \infty} \frac{1}{p} \|H_{\mathbb{R}}\|_{\infty, p},$$

and we conclude that $\lim_{p \rightarrow \infty} p^{-1} \|H_{\mathbb{R}}\|_{\infty, p} = 2(e\pi)^{-1}$.

We turn next to the complex case. What follows is a small variation of a construction used to prove vector-valued inequalities [3, pp. 311–315]. We begin by noting that for arbitrary real numbers λ_1 and λ_2 and $0 < p < \infty$, we have

$$\int_{\mathbb{R}^2} |x_1 \lambda_1 + x_2 \lambda_2|^p e^{-\pi|x|^2} dx = A_p^p (\lambda_1^2 + \lambda_2^2)^{p/2},$$

where

$$A_p = \left(\frac{\Gamma(\frac{p+1}{2})}{\pi^{\frac{p+1}{2}}} \right)^{1/p}.$$

if $f = f_1 + if_2$ is complex valued function with f_1 and f_2 are real valued, then

$$\begin{aligned} \|Hf\|_p^p &= \frac{1}{2\pi} \int_{\mathbb{T}} |(Hf_1)^2 + (Hf_2)^2|^{p/2} d\theta \\ &= \frac{A_p^{-p}}{2\pi} \int_{\mathbb{T}} \int_{\mathbb{R}^2} |x_1 Hf_1 + x_2 Hf_2|^p e^{-\pi|x|^2} dx d\theta \\ &= A_p^{-p} \int_{\mathbb{R}^2} \frac{1}{2\pi} \int_{\mathbb{T}} |H(x_1 f_1 + x_2 f_2)|^p d\theta e^{-\pi|x|^2} dx \\ &\leq A_p^{-p} \int_{\mathbb{R}^2} (\|H_{\mathbb{R}}\|_{\infty,p})^p \|x_1 f_1 + x_2 f_2\|_{\infty}^p e^{-\pi|x|^2} dx. \end{aligned}$$

Since $x_1 f_1 + x_2 f_2 = \operatorname{Re}[(x_1 - ix_2)(f_1 + f_2)]$, we get

$$\|Hf\|_p^p \leq A_p^{-p} (\|H_{\mathbb{R}}\|_{\infty,p})^p \|f\|_{\infty}^p \int_{\mathbb{R}^2} |x|^p e^{-\pi|x|^2} dx$$

and therefore

$$\|H\|_{\infty,p} \leq A_p^{-1} \left(\int_{\mathbb{R}^2} |x|^p e^{-\pi|x|^2} dx \right)^{1/p} \|H_{\mathbb{R}}\|_{\infty,p} = A_p^{-1} \pi^{-p/2} \Gamma(p/2 + 1) \|H_{\mathbb{R}}\|_{\infty,p}.$$

Using again Stirling's formula, we obtain

$$\lim_{p \rightarrow \infty} \frac{1}{p} \|H\|_{\infty,p} \leq \lim_{p \rightarrow \infty} \frac{1}{p} \|H_{\mathbb{R}}\|_{\infty,p} = \frac{2}{\pi e}.$$

Since obviously $\|H_{\mathbb{R}}\|_{\infty,p} \leq \|H\|_{\infty,p}$, we get the desired result. \square

Since

$$P_1^+ f(e^{i\theta}) = \frac{1}{2} (f(e^{i\theta}) + i\tilde{f}(e^{i\theta})) + \frac{1}{2} \hat{f}(0),$$

we have

$$\frac{\|H\|_{\infty,p}}{2} - 1 \leq \|P_1^+\|_{\infty,p} \leq 1 + \frac{\|H\|_{\infty,p}}{2},$$

and we therefore obtain Theorem 2 as an immediate consequence of Theorem 2'.

We note that since $\|P_n^+\|_{\infty,p} \leq \|P_n^+\|_{p,p}$, Hollenbeck and Verbitsky's theorem gives

$$\limsup_{p \rightarrow \infty} p^{-n} \|P_n^+\|_{\infty,p} \leq \pi^{-n}.$$

On the other hand, using the function $f(\zeta_1) \cdots f(\zeta_n)$ with f as in the proof of Theorem 2', we obtain

$$\liminf_{p \rightarrow \infty} p^{-n} \|P_n^+\|_{\infty,p} \geq (\pi e)^{-n}.$$

A. Chang and R. Fefferman's counterpart to the John–Nirenberg theorem (see [1]) could be used in place of Zygmund's lemma. However, we are not aware of any version of the John–Nirenberg theorem for \mathbb{T}^n that is sufficiently precise to improve our asymptotic estimates for $n > 1$.

4. CRITICAL EXPONENTS FOR $n > 1$

Theorem 4. *We have $2 + \frac{2}{2^n - 1} \leq p_n < 4$ when $n > 1$.*

Here the right inequality is of interest because it shows that $p_1 > p_2$, and this means that the problem is truly multi-dimensional in contrast to the one for the L^p to L^p constants. The left inequality is probably far from optimal; the main point of this estimate is the fact that we have $p_n > 2$ for every n . It appears to be a difficult problem to decide whether $\lim_{n \rightarrow \infty} p_n > 2$.

Proof of Theorem 4. We prove the left inequality by induction on n . By Theorem 1, this inequality is in fact an equality when $n = 1$.

Suppose now that we have $\|P_{n-1}^+\|_{\infty, q} = 1$ for $q = 2 + 2/(2^{n-1} - 1)$. Consider P_n^+ as the composition of the projections P_{n-1}^+ acting on the first $n-1$ variables and P_1^+ acting on the n -th variable. If we write $\zeta = (\xi, \zeta_n)$ with $\xi = (\zeta_1, \dots, \zeta_{n-1})$, we may write this as

$$P_n^+ f(\zeta) = P_{1, \zeta_n}^+ P_{n-1, \xi}^+ f(\xi, \zeta_n).$$

We now observe that since $\|P_1^+\|_{2,2} = \|P_1^+\|_{\infty,4} = 1$, the Riesz–Thorin theorem implies that we also have $\|P_1^+\|_{q,p} = 1$ when

$$2 < q < \infty \quad \text{and} \quad p = \frac{4q}{2+q}.$$

Setting

$$p = \frac{4q}{2+q} = 2 + \frac{2}{2^n - 1},$$

we therefore obtain

$$\begin{aligned} \|P_n^+ f\|_p^p &= \int_{\mathbb{T}^{n-1}} \int_{\mathbb{T}} |P_{1, \zeta_n}^+ P_{n-1, \xi}^+ f(\xi, \zeta_n)|^p d\sigma_1(\zeta_n) d\sigma_{n-1}(\xi) \\ &\leq \int_{\mathbb{T}^{n-1}} \left(\int_{\mathbb{T}} |P_{n-1, \xi}^+ f(\xi, \zeta_n)|^q d\sigma_1(\zeta_n) \right)^{p/q} d\sigma_{n-1} \\ &\leq \left(\int_{\mathbb{T}} \int_{\mathbb{T}^{n-1}} |P_{n-1, \xi}^+ f(\xi, \zeta_n)|^p d\sigma_{n-1}(\xi) d\sigma_1(\zeta_n) \right)^{q/p} \\ &\leq \left(\int_{\mathbb{T}} \left(\sup_{\xi \in \mathbb{T}^{n-1}} |f(\xi, \zeta_n)| \right)^p d\sigma_1(\zeta_n) \right)^{q/p} \leq \|f\|_\infty^q. \end{aligned}$$

We clearly have $p_{n+1} \leq p_n$, so the only remaining task is to show that $p_2 < 4$. Let g be a homogeneous holomorphic polynomial on the bidisk with $\|g\|_\infty \leq 1$. We set $h = (1-g)/(1-\bar{g})$ and find that

$$P_2^+ h = P_2^+ ((1-g)(1+\bar{g}+\bar{g}^2+\dots)) = 1 - P_2^+ (|g|^2) - g = 1 - \|g\|_2^2 - g.$$

It follows that

$$(P_2^+ h)^2 = (1 - \|g\|_2^2)^2 - 2(1 - \|g\|_2^2)g + g^2,$$

and since the functions $1, g, g^2$ are mutually orthogonal, we may compute $\|P_2^+ h\|_4$ explicitly:

$$\|P_2^+ h\|_4^4 = \|(P_2^+ h)^2\|_2^2 = 1 + \|g\|_2^8 + \|g\|_4^4 - 2\|g\|_2^4.$$

Thus $\|P_2^+ h\|_4 > 1$ whenever $\|g\|_4^4 \geq 2\|g\|_2^4$. One can take, for example, the polynomial

$$g(z_1, z_2) = \frac{(z_1 + z_2)^3}{10}.$$

□

Note that we may obtain a slightly better upper bound by computing the L^p norm of $P_2^+ h$ from the expansion of $(P_2^+ h)^{p/2}$ into a power series in g . By this approach, using the polynomial

$$g(z_1, z_2) = \frac{(z_1 + z_2)^{10}}{1025},$$

we have found that in fact $p_2 \leq 3.67632$.

5. THE CRITICAL EXPONENT FOR $n = \infty$

Let σ_∞ denote Haar measure on \mathbb{T}^∞ normalized so that $\sigma_\infty(\mathbb{T}^\infty) = 1$, and let $L^p(\mathbb{T}^\infty)$ be the corresponding L^p spaces. A multi-index α is now a sequence

$$\alpha = (\alpha_1, \alpha_2, \dots),$$

where only finitely many of the integers α_j are nonzero. We write $\alpha \geq \beta$ if we have $\alpha_j \geq \beta_j$ for every j .

The Riesz projection of a function f in $L^2(\mathbb{T}^\infty)$ with Fourier series

$$f(\zeta) = \sum_{\alpha} \hat{f}(\alpha) \zeta^\alpha,$$

can now be written as

$$P_\infty^+ f(\zeta) = \sum_{\alpha \geq 0} \hat{f}(\alpha) \zeta^\alpha.$$

We define the critical exponent of P_∞^+ as

$$p_\infty = \sup \{p \geq 2 : \|P_\infty^+\|_{\infty, p} = 1\}.$$

Note the following difference from the finite-dimensional case: we have either $\|P_\infty^+\|_{\infty, p} = 1$ or $\|P_\infty^+\|_{\infty, p} = \infty$, so that $\|P_\infty^+\|_{\infty, p} = \infty$ for $p > p_\infty$.

We want to show that $p_\infty = \lim_{n \rightarrow \infty} p_n$. It is clear that the limit exists and that $p_\infty \leq \lim_{n \rightarrow \infty} p_n$. To show that we have equality, we assume that $2 \leq p_\infty < \lim_{n \rightarrow \infty} p_n$. Let φ be a function of norm 1 in $L^\infty(\mathbb{T}^\infty)$ such that $\|P_\infty^+ \varphi\|_p > 1$ for $p_\infty < p < \lim_{n \rightarrow \infty} p_n$. Let n be a positive integer and set

$$\varphi_n(\zeta_1, \dots, \zeta_n) = \int_{\mathbb{T}^\infty} \varphi(\zeta_1, \dots, \zeta_n, \xi_{n+1}, \xi_{n+2}, \dots) d\sigma_\infty(\xi).$$

Then $\|\varphi_n\|_\infty \leq \|\varphi\|_\infty$. We observe also that

$$P_n^+ \varphi_n(\zeta) = P_\infty^+ \varphi(\zeta_1, \dots, \zeta_n, 0, 0, \dots).$$

It is plain that $\|P_n^+ \varphi_n\|_p \leq \|P_\infty^+ \varphi\|_p$. On the other hand, since $P_n^+ \varphi_n \rightarrow P_\infty^+ \varphi$ in L^2 , there is a subsequence $P_{n_k}^+ \varphi_{n_k}$ converging to $P_\infty^+ \varphi$ almost everywhere. Thus, by Fatou's lemma, $\|P_\infty^+ \varphi\| \leq$

$\lim_{k \rightarrow \infty} \|P_{n_k}^+ \varphi_{n_k}\|_p$. Hence $\lim_{n \rightarrow \infty} \|P_n^+ \varphi_n\|_p = \|P_\infty^+ \varphi\|_p$, which means that $\|P_n^+ \varphi_n\|_p > 1$ for sufficiently large n . This contradicts the assumption that $p < p_n$.

We conclude that if we could prove that $\lim_{n \rightarrow \infty} p_n > 2$, then we would have a positive answer to the question asked in the first paragraph of this note.

6. EXTENSIONS AND COMMENTS

The preceding results about critical exponents extend to the following more general setting. Let G be a compact abelian group and let E be a subset of the dual group \hat{G} . We then define the E -projection of a function f in $L^2(G)$ as

$$P_E f(\omega) = \sum_{\gamma \in E} \hat{f}(\gamma) \langle \gamma, \omega \rangle, \quad \omega \in G.$$

When E generates an order in the dual group \hat{G} (as it may for connected groups G), the proof of Theorem 1 still works, so that P_E has critical exponent 4. Observe also that a direct analogue of Theorem 4 can be obtained in this case.

In an attempt to simplify matters, we have studied the following example which appears to be the simplest nontrivial case, at least from a computational point of view. Take $G = \mathbb{Z}_3$, and consider Riesz projection to be the operator obtained by restricting to the set $\{0, 1\} \subset \mathbb{Z}_3$ in the Fourier domain. The set $\{0, 1\}$ does not generate an order, so we can not apply the observations made above. However, we may compute the critical exponent using the fact that the problem of maximizing the p -norm of the projection has the following geometrical interpretation. Indeed, we want to compute the maximum of $m_0^p + m_1^p + m_2^p$ where m_0, m_1, m_2 are the lengths of the medians of a triangle with vertices in the closed unit disc. It may be seen that the critical exponent equals the solution of the equation $2^p + 2 = 3(3/2)^p$, which means that $p_1 = 3.08164\dots$. It is a curious fact that this number is also the critical exponent of a different projection in [2, p. 265].

The corresponding multi-variable problem seems to be not much easier than the one for \mathbb{T}^n . Even for \mathbb{Z}_3^2 we have not been able to compute the critical exponent numerically. All we can say is that the critical exponent for $P_{\{0,1\}^2}$ is strictly smaller than p_1 and in fact $p_2 \leq 2.93039\dots$. This is far from the corresponding lower bound 2.28107... obtained from the Riesz–Thorin theorem, cf. the proof of Theorem 4.

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